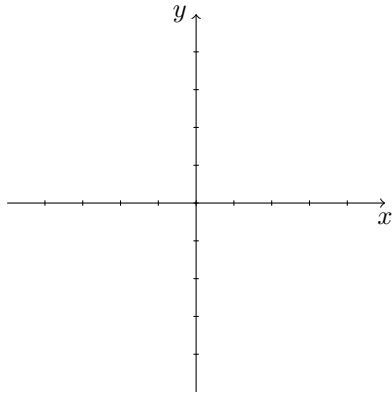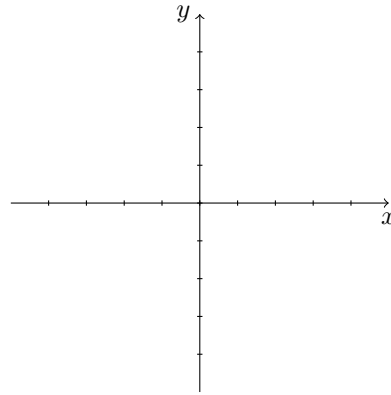# LECTURE: 1-3: NEW FUNCTIONS FROM OLD FUNCTIONS

Example 1: Using transformations, sketch graphs of the following functions. Include a sketch of the parent function as well as the final graph of the given function.
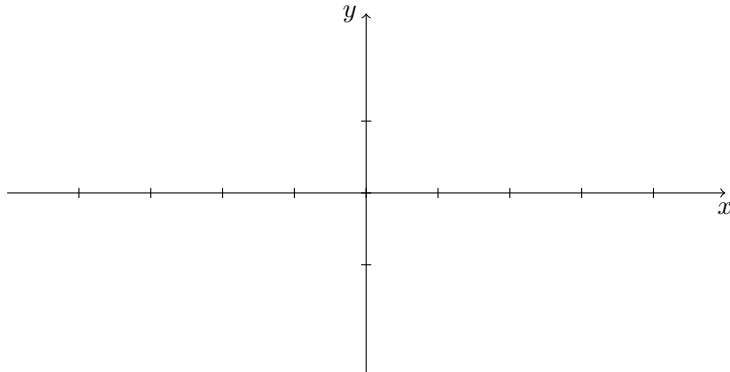
(a) $f(x) = \ln(x - 2) + 4$
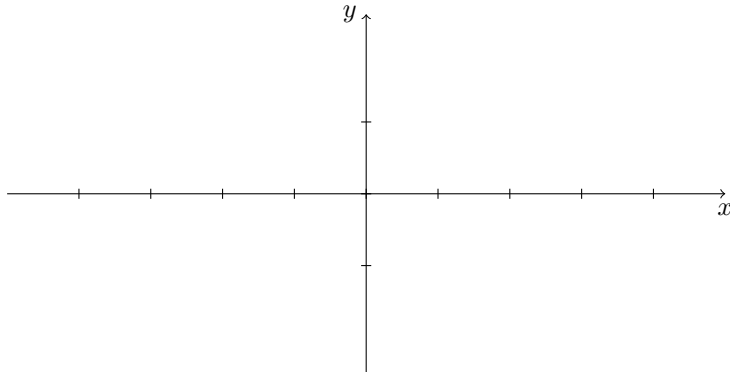
(b) $f(x) = e^{-x} - 3$

Example 2: Horizontal and vertical stretching and shrinking. Sketch graphs of the following functions on $[-2\pi, 2\pi]$. How do they relate to the parent function $f(x) = \sin x$?
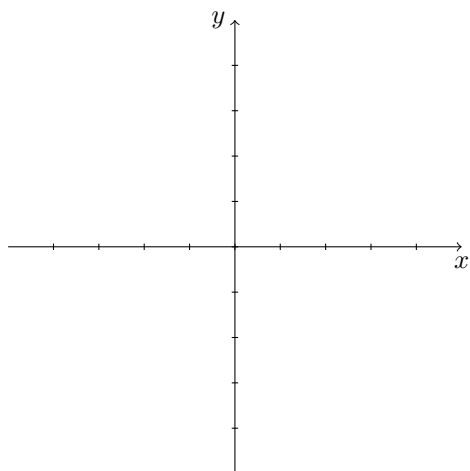
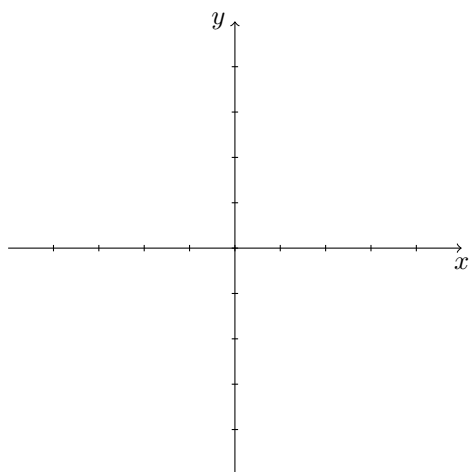(a) $g(x) = 2 \sin x$

(b) $h(x) = \sin(2x)$

Example 3: Review: completing the square and then using transformations. Use completing the square to write the following functions such that they can be graphed using transformations.

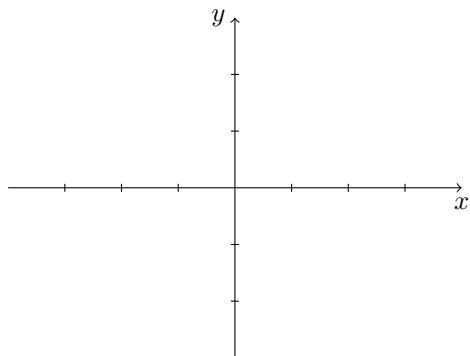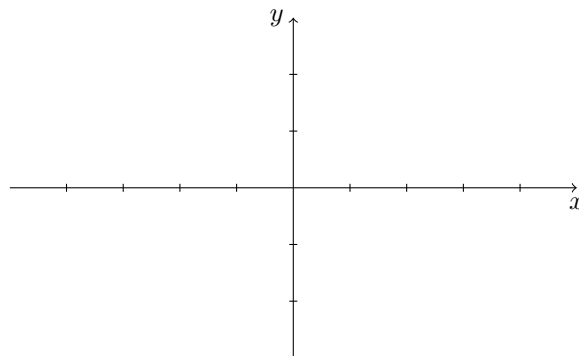(a) $f(x) = x^2 - 4x + 5$

(b) $f(x) = 4x - x^2$

Example 4: How to deal with absolute values. Sketch the graphs of the following functions:

(a) $y = |x^2 - 2|$

(b) $y = |\cos x|$

Combinations of Functions

Example 5: If $f(x) = \sqrt{x}$ and $g(x) = \sqrt{4 - x^2}$, find the following functions and their domains.

(a) $(f + g)(x)$

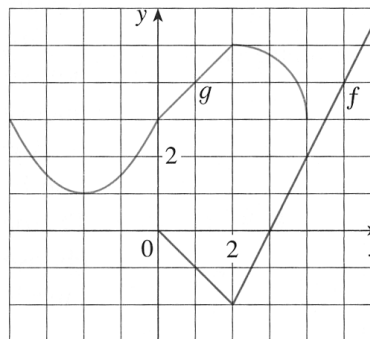(b) $(fg)(x)$

(c) $(f/g)(x)$

Composition of Functions

Given two functions f and g, the **composite function** $f \circ g$ is defined by

$$(f \circ g)(x) = f(g(x)).$$

Note: this is a **NEW** operation and is **NOT** the same as multiplying f and g.

Example 6: Use the graph below to find the following values or explain why it is undefined.

(a) $f(g(2))$

(b) $(g \circ g)(-2)$

Example 7: If $f(x) = x^2$ and $g(x) = x - 3$, find the composite functions $f \circ g$ and $g \circ f$. Is it true that $f \circ g = g \circ f$?

Example 8: If $f(x) = \cos x$ and $g(x) = 1 - \sqrt{x}$ find the following and their domains.

(a) $f \circ g$

(b) $g \circ f$

Example 9: Find $f \circ g \circ h$ if $f(x) = 2/(x+1)$, $g(x) = \cos x$ and $h(x) = \sqrt{x+3}$.

Example 10: What were those functions? Given the following compositions find, f, g and h such that $F = f \circ g \circ h$.

(a) $F(x) = \cos^2(x+9)$

(b) $F(x) = \tan^4(x^2 + 1)$

Example 11: Suppose g is an even function and let $h = f \circ g$. Is h also an even function?

Example 12: Let f and g be linear functions with equations $f(x) = m_1 x + b_1$ and $g(x) = m_2 x + b_2$. Is $f \circ g$ also a linear function? If so, what is the slope of its graph? What is its y-intercept?